Enrollment No: ______________________ **Exam Seat No:** ______________________

C. U. SHAH UNIVERSITY

Winter Examination-2019

Subject Name : Engineering Mathematics - I

Subject Code : 4TE01EMT2 **Branch: B.Tech (All)**

Semester : 1 **Date : 16/11/2019** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) If $y=\frac{1}{x}$ then y_n equal to

(A) $\frac{(-1)^n n!}{x^n}$ (B) $\frac{(-1)^n n!}{x^{n+1}}$ (C) $\frac{(-1)^{n-1}(n-1)!}{x^n}$ (D) none of these

b) If $y=e^{5x}\sin 3x$, then y_n equal to

(A) $(34)^{\frac{n}{2}} e^{5x}\sin\left(3x+n\tan^{-1}\frac{5}{3}\right)$ (B) $(34)^n e^{5x}\sin\left(3x+n\tan^{-1}\frac{3}{5}\right)$

(C) $(34)^{\frac{n}{2}} e^{5x}\sin\left(3x+n\tan^{-1}\frac{3}{5}\right)$ (D) none of these

c) If $y=\log(1+x)$, then x equal to

(A) $1+y+\frac{y^2}{2!}+\frac{y^3}{3!}$ (B) $1-\frac{y^2}{2!}+\frac{y^4}{4!}-.....$ (C) $y+\frac{y^2}{2!}+\frac{y^3}{3!}+\frac{y^4}{4!}+.....$

(D) None of these

d) The series $x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+.....$ represent expansion of

(A) $\cot^{-1}x$ (B) $\tan^{-1}x$ (C) $\sin^{-1}x$ (D) $\sin x$

e) $\lim_{x\to\infty} x^k e^{-mx}$ (k being a positive integer and $m>0$) = _____

(A) – 1 (B) 0 (C) 1 (D) None of these

f) $\lim_{x\to 0}\left(\frac{a^x+b^x}{2}\right)^{\frac{1}{x}}$ = _____

(A) $\log ab$ (B) $\sqrt{ab}$ (C) $\frac{1}{2}\log ab$ (D) none of these

g) If $x = r\cos\theta,\ y = r\sin\theta$ then $J\left(\frac{x,y}{r,\theta}\right)J'\left(\frac{r,\theta}{x,y}\right)$ is equal to

(A) 1 (B) – 1 (C) zero (D) none of these

h) If $u(x,y,z)=0$ then the value of $\frac{\partial x}{\partial y}\cdot\frac{\partial y}{\partial z}\cdot\frac{\partial z}{\partial x}$ is equal to

(A) 1 (B) – 1 (C) zero (D) none of these

i) If $f(x,y)=0$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{\partial f/\partial x}{\partial f/\partial y}$ (B) $\frac{\partial f/\partial y}{\partial f/\partial x}$ (C) $-\frac{\partial f/\partial y}{\partial f/\partial x}$ (D) $-\frac{\partial f/\partial x}{\partial f/\partial y}$

j) If $x = r\cos\theta,\ y = r\sin\theta,$ then

(A) $\frac{\partial x}{\partial r}=\frac{\partial r}{\partial x}$ (B) $\frac{\partial x}{\partial \theta}=0$ (C) $\frac{\partial x}{\partial r}=0$ (D) $\frac{\partial x}{\partial r}=\frac{1}{\partial r/\partial x}$

k) If $x_r = \cos\left(\frac{\pi}{2^r}\right)+i\sin\left(\frac{\pi}{2^r}\right)$ then $x_1x_2x_3\ldots\ldots$ to ∞ is

(A) – 3 (B) – 2 (C) – 1 (D) 0

l) The number of solutions to the equation $z^2+\bar{z}=0$ is

(A) 1 (B) 2 (C) 3 (D) 4

m) If the rank of matrix $\begin{bmatrix} l & -1 & 0 \\ 0 & l & -1 \\ -1 & 0 & l \end{bmatrix}$ is 2, then l equal to

(A) any column number (B) 3 (C) 1 (D) 2

n) A square matrix A is called orthogonal if

(A) $AA^{-1}=I$ (B) $A^2=A$ (C) $A^T=A^{-1}$ (D) $A^2=I$

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) If $y=\frac{1}{x^2+a^2}$ then find y_n. **(5)**

b) Prove that $e^x\sin x = x + x^2 + \frac{x^3}{3} - \frac{x^5}{30} - \frac{x^6}{90}\cdots\cdots$ **(5)**

c) If $V=\frac{1}{r}$ where $r^2=x^2+y^2+z^2$ then show that $V(x,\ y,\ z)$ satisfies Laplace's equation $\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}=0$. **(4)**

Q-3 **Attempt all questions** **(14)**

a) If $y=\sin\left(m\sin^{-1}x\right)$ then prove that **(5)**

$$\left(1-x^2\right)y_{n+2}-(2n+1)xy_{n+1}+\left(m^2-n^2\right)y_n=0.$$

b) Expand $f(x)=\frac{e^x}{e^x+1}$ in powers of x up to x^3 by Maclaurin's series. **(5)**

	c)	Evaluate: $\lim_{x\to 0}\left(\frac{a}{x}-\cot\frac{x}{a}\right)$	(4)
Q-4		**Attempt all questions**	**(14)**
	a)	Evaluate: $\lim_{x\to 0}\left(\frac{a^x+b^x+c^x+d^x}{4}\right)^{\frac{1}{x}}$	(5)
	b)	If $u=\frac{y^2}{x}$, $v=\frac{x^2}{y}$, evaluate $J=\left(\frac{x,y}{u,v}\right)$ and $J'=\left(\frac{u,v}{x,y}\right)$ and hence verify that $JJ'=1$.	(5)
	c)	Calculate approximate value of $\sqrt{9.12}$ by using Taylor's theorem.	(4)
Q-5		**Attempt all questions**	**(14)**
	a)	If $u=\tan^{-1}\left(\frac{x^2+y^2}{x-y}\right)$ then show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}\sin 2u$.	(5)
	b)	Evaluate: $\lim_{x\to 1}\left(\frac{x}{x-1}-\frac{1}{\log x}\right)$.	(5)
	c)	If $y=\frac{x^4}{(x-1)(x-2)}$ then find y_n .	(4)
Q-6		**Attempt all questions**	**(14)**
	a)	Using the formula $R=\frac{E}{I}$, find the maximum error and percentage of error in R if $I=20$ with a possible error of 0.1 and $E=120$ with a possible error of 0.05 and $R=6$.	(5)
	b)	Prove that $(a+ib)^{\frac{m}{n}}+(a-ib)^{\frac{m}{n}}=2(a^2+b^2)^{\frac{m}{2n}}\cos\left(\frac{m}{n}\tan^{-1}\frac{b}{a}\right)$.	(5)
	c)	Examine whether the following equations are consistent and solve them if they are consistent. $2x+6y+11=0,\ 6x+20y-6z+3=0,\ 6y-18z+1=0$	(4)
Q-7		**Attempt all questions**	**(14)**
	a)	Reduce the matrix $A=\begin{bmatrix}1&-1&2&-3\\4&1&0&2\\0&3&1&4\\0&1&0&2\end{bmatrix}$ to the normal form and find its rank.	(5)
	b)	Expand $\sin^5\theta\cos^2\theta$ in a series of sines of multiples of θ.	(5)
	c)	If $\tan(\alpha+i\beta)=x+iy$ then prove that $x^2+y^2+2x\cot 2\alpha=1$.	(4)
Q-8		**Attempt all questions**	**(14)**
	a)	Find the eigenvalues and eigenvectors of matrix $A=\begin{bmatrix}-3&-7&-5\\2&4&3\\1&2&2\end{bmatrix}$.	(5)
	b)	If $x_r=\cos\frac{\pi}{2^r}+i\sin\frac{\pi}{2^r}$ then prove that $\lim_{n\to\infty}x_1x_2x_3...x_n=-1$.	(5)

c) Verify Caley-Hamilton theorem for the matrix $A=\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$. **(4)**